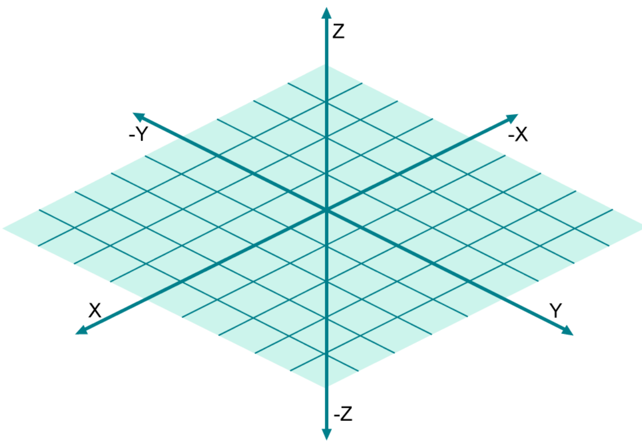
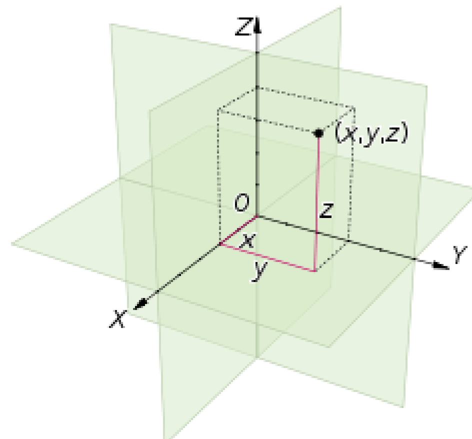
TOPIC 16-3: FORMULAS IN THREE DIMENSIONS

Space is the set of all points in three dimensions. Three coordinates are needed to locate a point in space. A three-dimensional coordinate system has 3 perpendicular axes: The x-axis, the y-axis, and the z-axis.

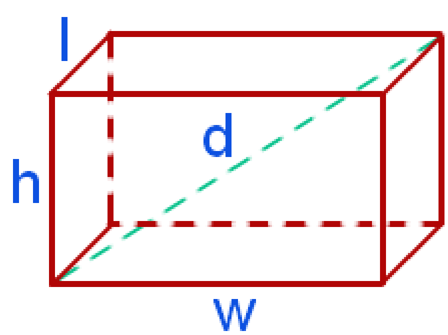




Formulas that we use in the two-dimensional coordinate system can also be extended to the three-dimensional coordinate system.

EXAMPLE 1: Diagonal of a right rectangular prism

The length of a diagonal, d, of a right rectangular prism with length, l, width, w, and height, h, is $d = \sqrt{l^2 + w^2 + h^2}$.



Find the length of the diagonal of a 3 in. by 4 in. by 5 in. rectangular prism.

EXAMPLE 2: Find the height of a rectangular prism with an 8 ft. by 12 ft. base and an 18 ft. diagonal.

EXAMPLE 3: The distance formula in three dimensions:

$$\boldsymbol{d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}}$$

Find the distance between the given points: (0, 0, 0) and (3, 4, 12)

EXAMPLE 4: The midpoint formula in three dimensions:

$$\boldsymbol{M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)}$$

Find the midpoint of the segment with the given endpoints:
(3, 8, 10) and (7, 12, 15)

A polyhedron is formed by four or more polygons that intersect only at their edges. Prisms and pyramids are polyhedrons, but cylinders and cones are not.

Using the shapes on the polyhedron activity, fill out the chart below for prisms. Look for a pattern and complete the chart.

Number of sides for the base	# of Vertices	# of Edges	# of Faces	V – E + F
3				
4				
5				
6				
7				
10				
n				

Using the shapes on the polyhedron activity, fill out the chart below for pyramids. Look for a pattern and complete the chart.

Number of sides for the base	# of Vertices	# of Edges	# of Faces	V – E + F
3				
4				
5				
6				
7				
10				
n				

Fill out the chart below for the regular polyhedrons (platonic solids) listed.

Name of regular polyhedron	# of Faces	# of Edges	# of Vertices	V – E + F
Tetrahedron (Triangular Pyramid)	4 triangles			
Cube	6 squares			
Octahedron	8 triangles			
Dodecahedron	12 pentagons			
Icosahedron	20 triangles			

What do you think is true about the relationship between the number of vertices, edges and faces of a polyhedron?